Assignment-I

Paper Name: Mathematics, Paper Code : M401 (CSE/IT-4th Sem)

1. If A and B are independent events prove that A^c and B^c are also independent events.

2. i) Urn I has 2 white and 3 black balls. Urn II has 4 white and 1 black and Urn III. 3 white and 4 black balls. An urn is selected at random and a ball drawn at random is found to be white. Find the probability that Urn I was selected.

   ii) An urn contains 4 white and 6 black balls. Two balls are successively drawn from the urn without replacement of the first ball. If the first ball is seen to be white, what is the probability that the second ball is also white.

3. A box contains 2000 components of which 5% are defective. A second box contains 500 components of which 40% are defective. Two other boxes contains 1000 components each with 10% defective components. We select at random one of the above boxes and remove from it at random a single component.
   (i) What is the probability that this component is defective? (ii) Finding that the selected component is defective, which is the probability that it was drawn from box 2?
   Ans. 0.1625, 0.615.
4. Two cards are drawn in succession from a pack of 52 cards. Find the chance that the first is a king and the second is a queen if the first card is (a) replaced (b) not replaced.
   Ans. 1/169, 4/663.
5. An Urn contains 10 white and 3 black balls, while another Urn contains 3 white and 5 black balls. Two balls are drawn from the first Urn and put in to the second Urn and then a ball is drawn from the latter. What is the probability that it is a white ball? Ans. 59/130.

6. The probabilities of A, B, C solving a problem are 1/3, 2/7 and 3/8 respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them will solve it.
   Ans. 25/56.
7. Three urns contain respectively 1 white and 2 black balls; 2 white and 1 black balls; 2 white and 2 black balls. One ball is transferred from the first to the second urn; then one ball is transferred from the second to the third urn; finally one ball is drawn from the third urn. Find the probability that the ball is white.
   Ans. 31/60.
8. A die is tossed thrice. A success is getting 1 or 6 on the toss. Find the probability distribution of the number of success.
9. Assuming that, on an average, 3 percent of the output of a factory making certain part is defective and that 300 units are in a package, what is the probability that at most five defective parts may be found in a package?

10. Show that $f(x) = x, \quad 0 < x < 1$
    $= k - x, \quad 1 < x < 2$
    $= 0, \quad$ otherwise
    is a density function for a suitable value of k. Calculate $P(1/3 < X < 1/2)$.

15. In a normal distribution, 31% of the item are under 45 and 8% are 64. Find the mean and S.D [Given that $P(0 < Z < 1.405) = 0.42$ and $P(-0.496 < Z < 0) = 0.19$]?

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16. The length of the bolts produced by the machine is normally distributed with mean 4 and S.D 0.5. A bolt is defective if its length does not lie in the interval (3.8, 4.3). Find the percentage of the defective bolts produced by the machine. $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0.6} e^{-\frac{t^2}{2}}\,dt = 0.725$

and $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0.4} e^{-\frac{t^2}{2}}\,dt = 0.6554$.

17. A forest contains 20 lions, of which 5 are captured tagged and then released. A few months later 4 of the 20 lions are captured. What is the probability that 2 of these 4 have been tagged?

18. The pdf of a random variable X is

$$f(x) = K(x-1)(2-x), \quad for\ 1 \le x \le 2.$$
$$= 0, \quad otherwise.$$

Determine (i) the value of the constant k ? ii) The distribution function F (X)?

19. Show that for the exponential distribution

$$f(x) = \frac{1}{\sigma} e^{\left(-\frac{x}{\sigma}\right)}, \quad 0<x<\infty,$$

mean and standard deviation both equal to σ.

20. The p.d.f. of a random variable is given by

$$f(x) = \begin{cases} kx+\frac{1}{2} & -1 \le x \le 1 \\ 0 & elsewhere \end{cases}$$

, find the value of *k* and distribution function (d.f).

(Ans. -1/2 < k <1/2)

21. Distribution of marks scored in an examination is normal. Samples of four students' marks are drawn and it is seen that the probability of the sample mean to be less than 61 is 0.44, to be more than 80 is 0.04. Find the mean and S.d of the distribution. [Given that P(0<z<a) = 0.06, 0.10, 0.46 according as a = 0.15, 0.25 and 1.75]

22. You have 100 bulbs whose lifetimes are independent exponentials with mean 5 hours. If the bulbs are used one at a time, with a failed bulb being replaced immediately by a new one, find the probability that there is still a working bulb after 525 hours?
(Given that P (0<Z<0.5)) = 0.1915.

23. In a certain factory blades are manufactured in packets of 10. There is a 0.2% probability for any blade to be defective. Using Poisson distribution calculates approximately the number of packets containing two defective blades in a consignment of 20,000 packets. ($e^{-0.02} = 0.9802$)

24. A random variable X has the following probability density function:

$$f(x) = \begin{cases} 2x, 0 \le x \le 1. \\ 0, \quad otherwise. \end{cases}$$

Find the value of Var(3- 5x) .

25. If X_n is mutually independent and identically distributed random variable with mean m and finite variable σ^2, and $S_n = X_1 + X_2 + \cdots + X_n$, then prove that WLLN (Weak Law of Large Numbers) holds for the sequence S_n .

26. In a sample of 120 workers in a factory, the mean and standard deviation of wages were Rs. 11.35 and Rs. 3.03 respectively. Find the number of workers getting wages between Rs. 9 and Rs. 17 in the whole factory, assuming that the wages are normally distributed. (Area under standard normal curve from z = 0 to z = 0.78 is 0.2823 and to z = 1.86 is 0.4686)

27. A bag contains 8 red balls and 5 white balls. Two successive draws of 2 balls are made without replacement. Find the probability that the first drawing will give 2 white balls and the second 2 red balls.

28. The weight of students in a college is normally distributed with mean 40kg and standard deviation 5kg. Find the percent of the students that have weight

i) Greater than 50 kg ii) Between 38 kg and 52 kg,

[Given that $\phi(2) = 0.9772,\ \phi(0.4) = 0.6554, \phi(2.4) = 0.9918$]

29. Show by Tchebyshiv's inequality that if a die is thrown 3,600 times, the probability of number of sixes lies between 550 and 650 is at least $\frac{4}{5}$?

30. Examine whether function |x| in (-1,1) and zero elsewhere is a density function.